

Virtual Graphs for High-Performance Embedded Topologies

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Summary

Many optimization and machine learning algorithms are commonly described as graph problems. For example, graphical models are often used to analyze the flow of traffic between cities or the transmission of information between neurons in an artificial neural network.

D-Wave quantum processing units (QPUs) solve graphical models—specifically, Ising minimization problems on a physical working graph made up of qubits and couplers. The new *virtual graphs* feature of the D-Wave 2000Q™ system provides users with improved embedding performance wrapped in a simplified interface. We describe the key enabling processor technologies, and provide a simple example with performance results enabled by this new feature in the D-Wave 2000Q system.

Embedding An Ising minimization problem is defined on a graph of *vertices* and *edges*. D-Wave QPUs solve Ising minimization problems using a hardware implementation of the quantum annealing algorithm, with qubits representing the input model's vertices and couplers representing the input model's edges. The weights of qubits and couplers that define an input are programmable, but the working graph of a QPU, i.e., which pairs of qubits are connected by couplers, is static.

If the graph of the input problem cannot be mapped directly to the working graph of the QPU, the input must be mapped using an *embedding* (see Figure 1). In an embedding, chains of physical qubits are coupled tightly together using strong ferromagnetic couplings so that all qubits in the chain are compelled to take the

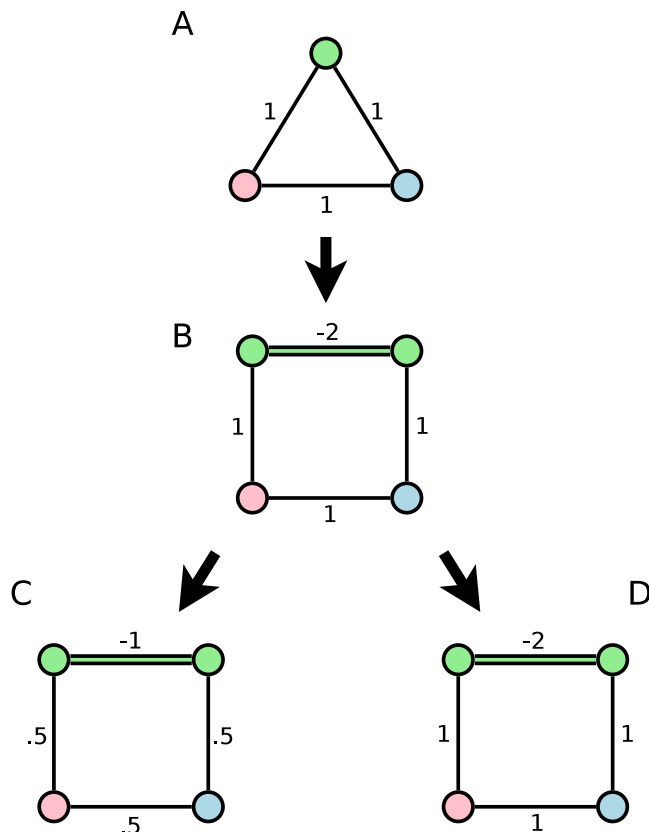


Figure 1: Embedding an input that does not fit directly on a D-Wave QPU. The original problem (A) has a green vertex which is replaced by a chain of two vertices connected with a ferromagnetic coupling (B). With the standard coupling range, the embedded problem must be scaled down for the QPU, which can lead to decreased performance (C). However, with the extended coupling range given by the new virtual graph features, no rescaling is necessary (D).

same value and the chain acts as a single logical qubit. Embedding allows logical qubits that have higher and more flexible connectivity than single logical qubits.

With previous D-Wave products, users had to perform embedding manually. However, the virtual graphs feature allows users to create virtual Ising model solvers for their desired graph—the entire process of creating, optimizing, and using an embedding is handled automatically, making the QPU much easier to use.

Energy scale While embedding allows D-Wave’s QPUs to solve inputs with arbitrary connectivity, it does have drawbacks compared to running inputs mapped directly to the QPU’s working graph. To ensure the chains act as logical qubits, the chain couplings must be strong compared to the input couplings between the chains. Since the range of coupling strengths available is finite, this is typically accomplished by setting chain couplings to the largest allowed negative value and scaling down the input couplings. Programming input couplings with a reduced energy scale can make it harder for the QPU to find global optima.

To mitigate this issue, we increased the available coupling range from $[-1, 1]$ to $[-2, 1]$. Since embedded problems typically have chain couplings that are at least twice as strong as the other couplings, and standard chain couplings are all negative, this effectively doubles the energy scale available for embedded problems; a simple example is shown in Figure 1.

We also provide users access to a new parameter that offsets the flux bias of a qubit. This parameter allows calibration adjustments to ensure chain behavior is unbiased, and also reduces the analog error associated with embedding.

Optimization The inputs that see the greatest benefits from virtual graphs are those that require both strong chains and high precision. To demonstrate the benefits of the new features, we generated random, high precision Ising model inputs on fully connected graphs. These inputs require long, strong chains—at the largest problem size, each chain consists of 17 physical qubits with coupling values of -8 .¹ For each input we gathered 10,000 samples from two D-Wave 2000Q QPU solvers: one with the virtual graphs feature on and one

¹Couplings of -8 will be scaled down to -2 with the virtual graphs feature and -1 without. The rest of the Hamiltonian must also be scaled down by factors of 4 and 8 respectively.

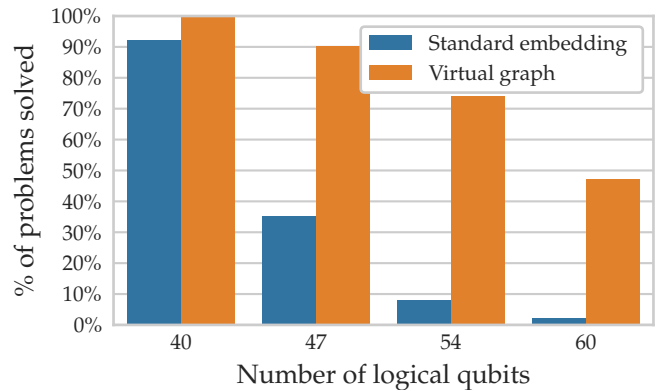


Figure 2: Percentage of high-precision fully connected Ising models for which a global optimum was found by the D-Wave 2000Q at least once in 10,000 anneals. The virtual graphs feature improves performance significantly, allowing the D-Wave QPU to find optimal states where it previously could not.

with it off. We measured the percentage of problems at each size for which each solver was able to find the unique global optimum; results are shown in Figure 2. At the largest problem size, the QPU found the optimum in 47% of the inputs with the virtual graphs feature turned on, as opposed to only 2% without.

Sampling Virtual graphs help with sampling as well as with optimization. Algorithms for training machine learning models often require sampling from Boltzmann distributions; Boltzmann distributions are defined with a temperature parameter and sampling from colder distributions is both more powerful and more computationally difficult. In diagnostic tests, the new virtual graphs feature improves the sampling temperature of D-Wave 2000Q QPUs by about 50%, allowing improved machine learning models to be trained and, more generally, opening the door to more powerful hybrid algorithms that can run on embedded inputs.

Conclusion Embedding inputs that do not fit directly on D-Wave QPUs will be a topic of increasing focus as qubit counts continue to grow at a steady exponential rate. The new virtual graphs feature allows chains representing logical qubits to behave more like native qubits on the working graph. This improves performance of embedded problems in both optimization and sampling.